Covariance Reconstruction for Track Fusion with Legacy Track Sources

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The problem of track-to-track association and track fusion has been considered in the literature where the fusion center has access to multiple track estimates and the associated estimation error covariances from local sensors, as well as their crosscovariances. Due primarily to the communication constraints in real systems, some legacy trackers may only provide the local track estimates to the fusion center without any covariance information. In some cases, the local (sensor-level) trackers operate with fixed filter gain and do not have any self assessment of their estimation errors. In other cases, the network conveys a coarsely quantized root mean square (RMS) estimation error of each local tracker. Thus the fusion center needs to solve the track association and fusion problem with incomplete data from legacy local trackers. The problem of track fusion with legacy track sources which lack covariances is handled by reconstructing them using sensor covariance and target maneuvering index information and then using the appropriate association and fusion algorithms. The situation when a coarsely quantized RMS estimation error is available is also discussed. A two-sensor tracking example is used to illustrate the effectiveness of the proposed covariance reconstruction method for track fusion and compared with a centralized interacting multiple model estimator.

Manuscript received February 9, 2007; revised August 14, 2008; released for publication October 23, 2008.

Refereeing of this contribution was handled by Benjamin Slocumb.

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1. INTRODUCTION

In multisensor target tracking, each sensor can have its own target state estimate based on the local sensor measurements. Most existing communication networks between local trackers/sensors transmit to a fusion center the local track estimates—sometimes without any estimation error covariances, sometimes with partial covariance information and only rarely with full covariance information. In order to form a global picture of the existing tracks, it is necessary to associate multiple local tracks and fuse them to obtain the global target state estimates. Under this tracking configuration, the fusion center can carry out this association and fusion of the (latest) local track estimates on demand, which, in general, is less frequent than the measurement rate at each local sensor. Another important reason that track fusion (TrkF) is a viable alternative to centralized tracking (CenT), which requires transmission of all the measurements to the fusion center, is that the performance of TrkF is very close to that of CenT [4]. The problem of associating tracks represented by their local state estimates and covariances from multiple sources has been studied extensively in literature. While different sensors typically have independent measurement errors, the local state estimation errors for the same target are dependent due to the common process noise (and the prior, if common). This dependence is characterized by the crosscovariances of the local estimation errors [3]. Methods have been proposed to fuse the local tracks that carry out decorrelation [11, 12, 13]. Other techniques include track fusion that explicitly utilizes the crosscovariance information in a Bayesian setting [7, 10], with asynchronous sensors [1], and more generally, with possible common priors [15, 16, 17]. The work of [20] dealt with simultaneous general track-to-track association and bias estimation. In addition, the "covariance intersection" method proposed in [14] can fuse two estimates with unknown correlation. However, it is a very conservative method that avoids the issue of crosscovariances but may yield a fused covariance with diagonal elements that indicate a degradation in each component from the best estimate before fusion [9].

A legacy sensor and tracking system is one that was built in the past under different requirements, specifically, with no requirements to support network fusion. Thus no hardware/software facilities (or inadequate facilities) were included in the system to support the kind of track fusion that is desired now. To get the relevant data that one would like out of the system (i.e., covariances) requires a significant hardware/software modification to the system, which is impractical. Concisely, legacy can be defined as "you are stuck with what you've got." Before fusing local tracks, the fusion center has to decide whether they are from the same target. Track association is a hypothesis testing problem where local tracks are considered as having common origin (from the same target) vs. different ones by comparing a certain test statistic with a threshold to obtain desired test power [5, 18]. However, no previous results are available for the association and fusion of local tracks with legacy trackers that do not provide the necessary covariance information of the estimation errors.

In this paper we first consider the approximation of the covariance of the estimation error from a legacy tracker with a fixed filter gain. Then we use a two-sensor tracking scenario to compare the performance of the track fusion algorithm with the centralized target state estimator where the fusion center uses the state-of-the-art interacting multiple model (IMM) algorithm. Both the estimation accuracy and the credibility (consistency [2]) of the distributed tracker are compared with those of the centralized one. The results indicate that the performance degradation is small even during target maneuvers.

The rest of the paper is organized as follows. Section 2 describes the model used for legacy track sources. Section 3 presents a method to obtain the covariance of a legacy filter's track estimate as well as an approximation of the crosscovariance between two tracks. The reconstruction of the track covariance from a coarsely quantized estimation RMS error is also discussed. Section 4 presents a tracking example where two distributed tracking configurations are compared with the centralized estimator. Concluding remarks are provided in Section 5.

2. LEGACY TRACK SOURCES

In this section the model used for legacy track sources is formulated assuming the trackers are Kalman filters. To simplify the discussion, the model is presented for one generic coordinate with the target motion given by a discretized continuous time white noise acceleration (DCWNA) model [2]. For asynchronous sensors this model should be used to consistently handle the white process noise for all values of the sampling interval.

For sampling interval T, the state and measurement equations are

$$x(k+1) = Fx(k) + v(k) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + v(k) \quad (1)$$

$$z(k) = Hx(k) + w(k) = [1 \quad 0]x(k) + w(k) \quad (2)$$

where $v(k)$ is the zero mean white process noise sequence with covariance

$$E[v(k)v(k)'] \stackrel{\Delta}{=} Q(t_{k+1} - t_k) = Q(T) = \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix} \tilde{q} \quad (3)$$

where $\tilde{q}$ is the (continuous time) process noise power spectral density (PSD)[1] and $w(k)$ is zero mean white measurement noise sequence, uncorrelated with the process noise, with variance

$$E[w(k)^2] = \sigma_w^2. \quad (4)$$

This describes the target motion along one dimension. For target motion in 2 or 3 dimensions, the model will consist of 2 or 3 such models with an appropriate stacked state vector.

The target maneuvering index, subscripted by "c" to indicate that it is based on the continuous time process noise [2], is defined as

$$\lambda_c = \sqrt{\frac{\tilde{q}T^3}{\sigma_w^2}}. \quad (5)$$

Then the steady state filter gain is

$$W = \begin{bmatrix} \alpha & \frac{\beta}{T} \end{bmatrix}' \quad (6)$$

where

$$\alpha = \beta\sqrt{u} \quad (7)$$

$$\beta = \frac{12}{6\left(u + \sqrt{u}\right) + 1} \quad (8)$$

$$u = \frac{1}{3} + \sqrt{\frac{1}{12} + \frac{4}{\lambda_c^2}}. \quad (9)$$

The steady state solution for the state estimation covariance matrix is given by

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} \alpha & \frac{\beta}{T} \\ \frac{\beta}{T} & \frac{\beta(\alpha - \beta/2)}{(1-\alpha)T^2} \end{bmatrix} \sigma_w^2. \quad (10)$$

The above solution is valid for the steady state of the DCWNA filter, but only with the *optimal* values of α and β as given in (7)–(8).

A legacy tracker uses a fixed gain W, not necessarily the optimal one, in each of its α-β filter updates and sends the state estimates to the fusion center, typically, without covariance information. Since track association and track fusion algorithms require such information in order to combine local tracks from different sources, a procedure to obtain this missing information is discussed next.

3. APPROXIMATION OF THE ESTIMATION ERROR COVARIANCE AND CROSSCOVARIANCE

Because of the time-varying target-sensor geometry, an α-β filter, even though it uses fixed gains, is not necessarily in steady state. This is due to the nonstationarity of the measurement noises, which is accounted for in

[1] See [2] on why it is incorrect to call this the variance of the process noise.

Subsection 3.1. Our model will assume that the tracking filter has a "slowly varying" (quasi-) steady state. The covariance of the target state estimate will be evaluated accounting for the fact that the sensor measurements (typically in polar or spherical coordinates for a radar), while having uncorrelated measurement noises between their components (range, azimuth/crossrange), have a coupling (correlation) between the track state estimation errors in different Cartesian coordinates. Subsection 3.2 deals with the case where the communication network provides partial covariance information in addition to the state estimates. A procedure to reconstruct the full state covariance matrix is presented. Since in the real world multiple sensors are practically never operating in a synchronized manner, the procedure for track fusion from asynchronous sensors is discussed in Subsection 3.3. Subsection 3.4 presents a simple method to approximate the crosscovariances between the state estimation errors of two local tracks from the same target by assuming constant correlation coefficients, whose exact values are shown to vary relatively little over the practical range of target maneuvering indices.

The hypothesis testing for track association and the fusion equations with the crosscovariance can be found in [3, Sec. 8.4].

3.1. Coupling Between Coordinates and Nonstationarity

For tracking in more than one dimension of the measurement space, the measurement covariance can be converted from the sensor coordinates (typically polar or spherical) into the coordinates in which the state is defined (usually Cartesian).[2] This will result in correlation between the state estimation errors in the Cartesian coordinates. It is important to preserve the coupling between the coordinates when the uncertainty ellipse for position is elongated and slanted, e.g., a "cigar" with the main axes at 45° and 135°. Neglecting the correlation between the coordinates would yield a much larger uncertainty region.

To preserve the coupling between the state space coordinates due to the measurements, the fusion center should run the Joseph form of the covariance update iteration[3] at time k [2]

$$P(k) = [I - WH][FP(k-1)F' + Q][I - WH]' + WR(k)W' \tag{11}$$

with the appropriate sampling interval. The Joseph form is needed because the legacy filter gain is not optimal and only this equation is valid for the covariance (actually MSE matrix) update when *arbitrary* filter gains are used. The process noise covariance Q should be selected by the fusion center to model the target motion uncertainty to the extent possible. The filter gain W in (11) should be the same as in the legacy filter. If W is not known at the fusion center, it should be "replicated" using (6). The measurement noise covariance $R(k)$, assumed to be known,[4] in (11) is the covariance of the measurements converted from polar to Cartesian. The measurement conversion should be linearized at the latest measurement or the measurement prediction using the latest state. When $P(k-1)$ is unavailable at the fusion center, one can assume that $P(k-1) = P(k)$ in (11), resulting in an algebraic Riccati equation. This will yield a (slowly) time-varying covariance matrix that accounts for the nonstationarity of measurement noise.

3.2. Approximation of the Estimation Error Covariance of Legacy Trackers with Partial Information

When the communication network can provide a coarsely quantized (i.e., an approximate) 2-dimensional root mean square (RMS) position error, denoted as RMS_p, to the fusion center, the state estimation error covariance can be obtained as follows.

We shall model RMS_p as the (steady state) error of two independent α-β filters, one in the range direction, the other in the cross-range direction. Denote the measurement noise RMS values in these directions as σ_r and $\sigma_{\times r}$, respectively. These are assumed to be known, based on the radar specifications and the radar-target geometry.

The position gains for these two filters are, according to (6),

$$\alpha_r = \alpha(\lambda_{c_r}) \tag{12}$$

and

$$\alpha_{\times r} = \alpha(\lambda_{c_{\times r}}) \tag{13}$$

respectively, where the corresponding target maneuvering indices are, similarly to (5),

$$\lambda_{c_r} = \sqrt{\frac{\tilde{q}T^3}{\sigma_r^2}} \tag{14}$$

and

$$\lambda_{c_{\times r}} = \sqrt{\frac{\tilde{q}T^3}{\sigma_{\times r}^2}} \tag{15}$$

with $\tilde{q}$ the continuous time process noise PSD that models the motion uncertainty (in both the range and cross-range directions, uncorrelated between them) and T the sampling interval.

The RMS position error from the above filters is, based on (10), given by

$$\mathrm{RMS}_p = \sqrt{\alpha_r\sigma_r^2 + \alpha_{\times r}\sigma_{\times r}^2}. \tag{16}$$

[2] While some tracking systems keep the measurements in polar/spherical coordinates, the conversion to Cartesian allows exact debiasing when necessary [2].

[3] A single time argument is used here for the covariance.

[4] The measurement noise variances are, particularly in azimuth/elevation (and thus in cross-range), dependent on the target SNR (inversely proportional to the SNR [6]; in range the variance depends primarily on the pulse waveform). However, unless one assumes these variances as known (for an "average" SNR), one cannot reconstruct the track errors. Consequently, $R(k)$ is assumed known.

Assuming the value of RMS_p is available and the measurement noise variances are known (as discussed previously), one can solve (16) (after substituting (14)–(15) into (12)–(13) and the result into (16)) to find $\tilde{q}$. Once this is obtained, one can use (10) or (11) to reconstruct (approximately) the covariance of the entire state estimate. Note, however, that while this is in a Cartesian coordinate system, this system is aligned with the line of sight from the radar to the target and it has to be rotated into the local (common) Cartesian system, which is, typically, East-North.

The above procedure allows to reconstruct (approximately) the estimation error covariance of the legacy tracker from a coarsely quantized position RMS error, which is assumed to be conveyed by a communication network. A similar approach can be taken when RMS_p is a position prediction error, as well as for the 3-dimensional case.

3.3. Prediction to Fusion Time for Asynchronous Sensors

For asynchronous sensors, the state prediction (to the time for which fusion will be carried out) based on the legacy tracker's latest estimate should be used by the fusion center. Assume that the fusion is done at time k and the most recent estimate at the fusion center from the legacy tracker is $\hat{x}(\kappa)$ at time[5] κ, with $\kappa < k$. Then the fusion center needs to (i) approximate the estimation error covariance[6] $P(\kappa)$ at time κ using (10) or (11) and (ii) apply the standard prediction equations given by

$$\hat{x}(k) = F(k,\kappa)\hat{x}(\kappa) \tag{17}$$

$$P(k) = F(k,\kappa)P(\kappa)F(k,\kappa)' + Q(k,\kappa) \tag{18}$$

to obtain the state prediction and the corresponding error covariance for time k. For the motion model (1), $Q(k,\kappa)$ is given by (3) with $T = t_k - t_\kappa$.

Thus what is needed to evaluate the covariance of the estimate from a legacy tracker are:

- the sampling times
- the process noise PSD
- the measurement noise covariance.

It should be noted that the parameters based on which the legacy tracker has been designed are unlikely to be the same as listed above. Thus, what the fusion center should do is to replicate the performance of the legacy tracker to the extent possible.

[5] We use for simplicity the notations κ and k instead of t_κ and t_k.

[6] A single time argument is used here for the covariance. This covariance can be an updated covariance at the current time for one sensor, or a prediction to the current time for another sensor.

3.4. Approximation of the Crosscovariance of the Estimation Errors

When two local tracks have correlated estimation errors, assuming they are operating synchronously[7] and use the same target motion and measurement models, in the steady state, the crosscovariance matrix is given by [3]

$$P^{\times} = [I - WH][FP^{\times}F' + Q][I - WH]'. \tag{19}$$

The above Lyapunov type matrix equation can be solved numerically for any given target maneuvering index by simple forward iteration starting from $P^{\times} = 0$. For a distributed tracking system, the calculation of the crosscovariance using (19) is not practical.

The following approximation is considered [8]. Denote by P^{ij} the approximate crosscovariance matrix between local tracks i and j. Each element of P^{ij}, which is a 2×2 matrix for the model considered in (1), is approximated by constant correlation coefficients as follows

$$P^{ij}_{lm} = \rho_{lm}[P^i_{ll}P^j_{mm}]^{1/2}, \qquad l,m = 1,2 \tag{20}$$

where ρ_{11} is the position-position correlation coefficient, ρ_{12} is the position-velocity correlation coefficient and ρ_{22} is the velocity-velocity correlation coefficient.

Assuming equal variances of the measurement error for both sensors, we can solve the Lyapunov equation for the steady state DCWNA model. The resulting crosscorrelation coefficients between the estimation errors from the two local trackers, namely, ρ_{11}, ρ_{22} and ρ_{12}, are shown in Fig. 1 for target maneuvering index values within $[0.05, 2]$. These results are similar to those in [8] where the discrete time white noise acceleration model (DWNA) [2] is used. For the simulations to be presented in Section 4, we choose, in view of the fact that, as it can be seen from Fig. 1, these coefficients are nearly constant, the following fixed values

$$\rho_{11} = 0.15, \qquad \rho_{12} = 0.25, \qquad \rho_{22} = 0.7 \tag{21}$$

to compute the approximate crosscovariance according to (20).

Legacy trackers can be assumed as being decoupled across coordinates since the process noise is assumed uncorrelated between different coordinates. Consequently, the crosscorrelation between one sensor's tracking errors in one coordinate and another sensor's tracking errors in another coordinate will be zero due to the lack of common process noise. Thus the crosscovariance matrix will be assumed to have blocks consisting of zeros between different coordinates.

[7] The exact general recursion of the crosscovariance for asynchronous sensors is presented in the Appendix.

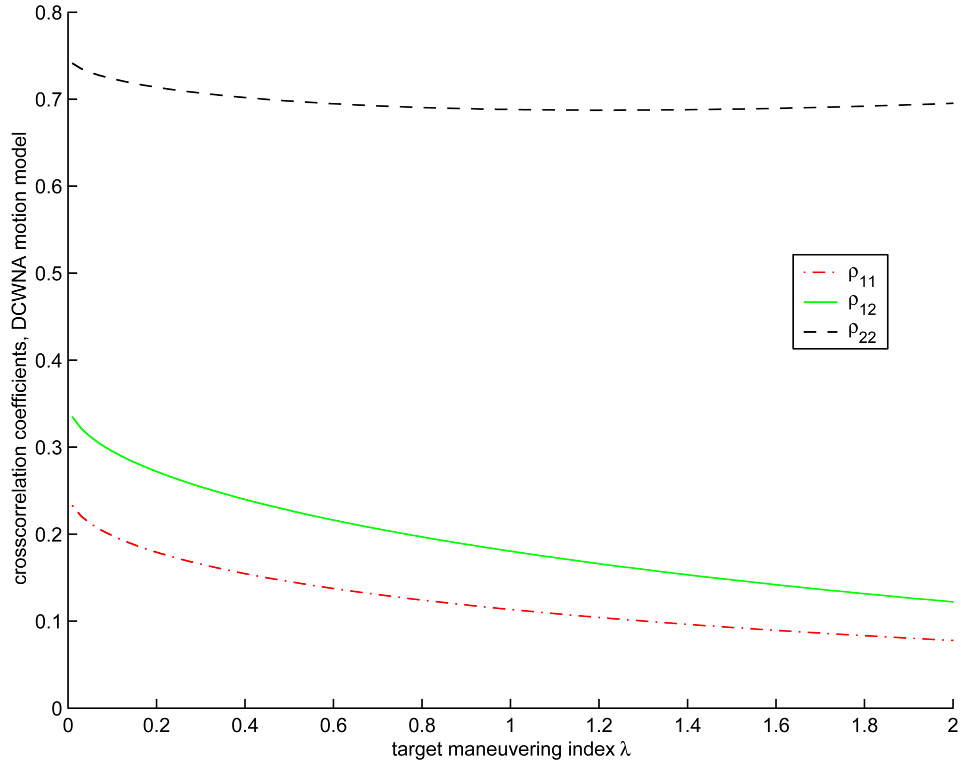

Fig. 1. Crosscorrelation coefficients vs. target maneuvering index for DCWNA model.

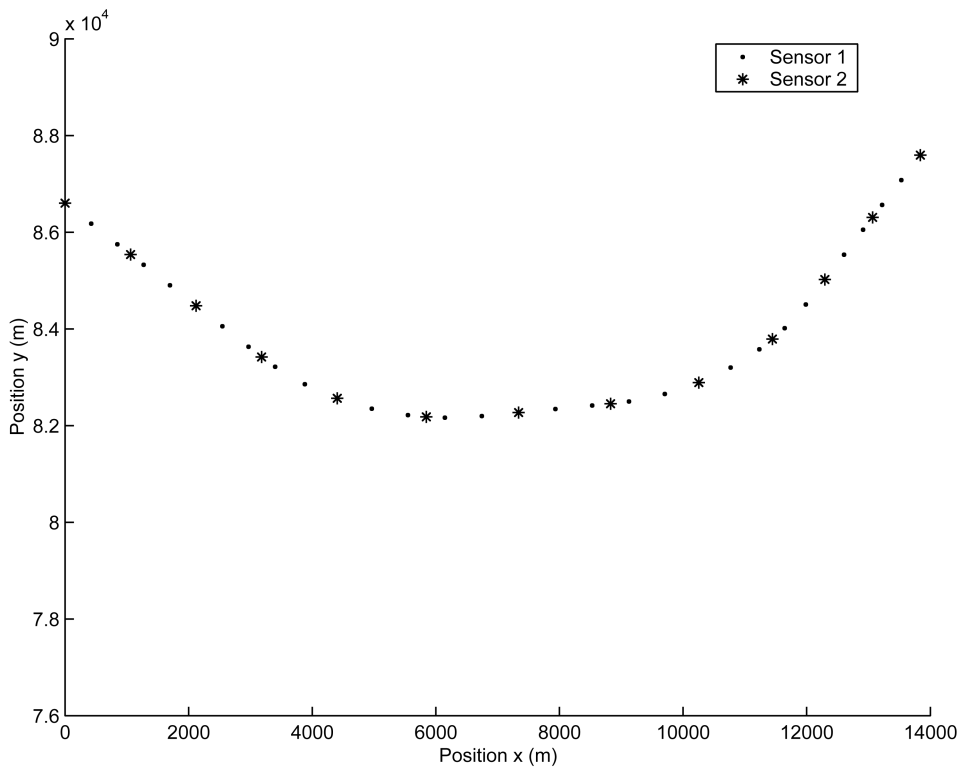

Fig. 2. Target trajectory with true positions where measurements are made by the two sensors.

4. EXAMPLE OF TRACK FUSION WITH A LEGACY TRACK SOURCE

We consider a ground target tracking scenario where two sensors are located at $(-50,0)$ km and $(50,0)$ km, respectively. Both sensors measure the target range and bearing with the same standard deviations of the measurement error given by $\sigma_r = 50$ m and $\sigma_b = 2$ mrad. The sampling interval of sensor 1 is $T_1 = 2$ s while the sampling interval of sensor 2 is $T_2 = 5$ s.

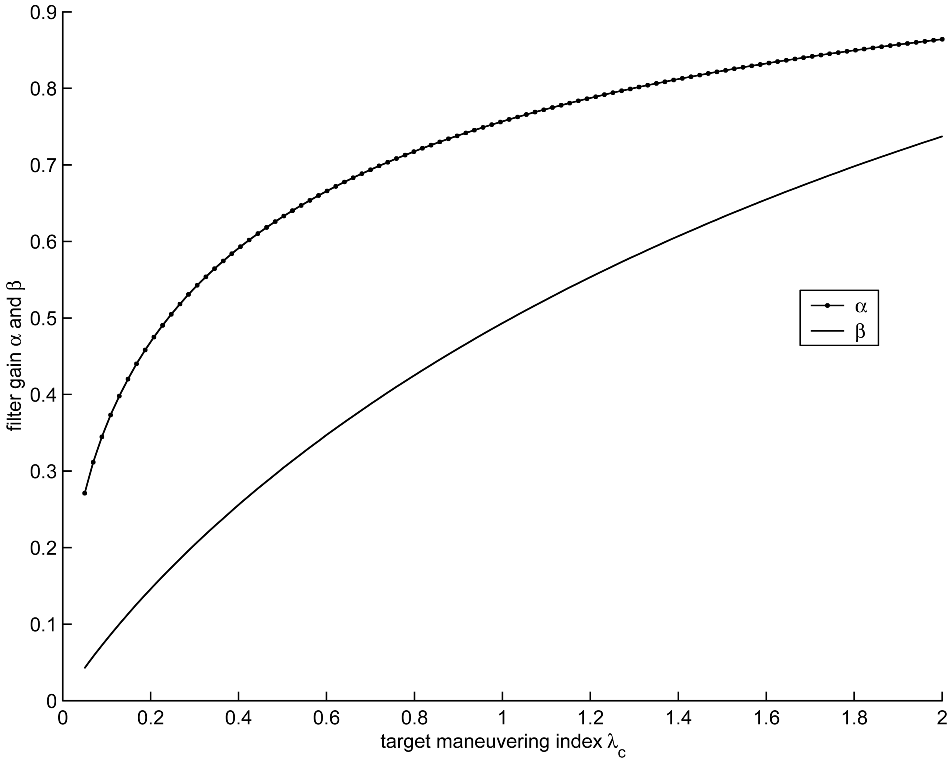

Fig. 3. Steady state filter gains vs. target maneuvering index for DCWMA model.

The target is initially at (0, 86.6) km moving at a speed of 300 m/s toward south-east on a course of approximately $-135°$. Then at $t = 15$ s the target makes a course change with a constant turn rate of 4°/s (acceleration of about 2.1 g over a duration T_{man} of about 11 s) and heads toward east. The target makes a second course change at $t = 35$ s with a constant turn rate of 4°/s and heads toward north-east. The target trajectory is shown in Fig. 2 where the true target positions are indicated at the time instances at which a measurement is made by sensor 1 or sensor 2. The total time for the target to complete the designated trajectory is 60 s. Note that the target range is around 100 km at the beginning for both sensors, where the standard deviation of the crossrange measurement error is around 200 m. The true target motion has no process noise in this case.

We consider the following three tracking configurations for performance comparison.

(i) A centralized estimator which uses an IMM with two models and sequentially updates the target state with measurements from both sensors. This IMM estimator has a DCWNA model with low process noise PSD $\tilde{q}_l$ to capture the uniform target motion and a DCWNA model with high process noise PSD $\tilde{q}_h$ to capture the two turns. We use $\tilde{q}_l = 1$ m^2/s^3 and $\tilde{q}_h = 8000$ m^2/s^3 which, for $T_1 = 2$ s, corresponds to a target maneuvering of $\sqrt{\tilde{q}_h/T_1} \approx 6.4$ g. The process noise is the same in east and north of the Cartesian coordinates and uncorrelated between these coordinates. The transition between the modes is modeled according to a continuous time Markov chain with the expected sojourn times [2, Sec. 11.7.3] in these modes given by $1/\lambda_1$ and $1/\lambda_2$, respectively. These correspond to exponential sojourn time distributions with parameters λ_1 and λ_2, respectively. The transition probability matrix between the two models (generalized version of Eq. (11.6.7-1) in [2]) from any time t_1 to time t_2 is [19]

$$\Pi(t_2,t_1) = \frac{1}{\lambda_1+\lambda_2}\begin{bmatrix} \lambda_2 + \lambda_1 e^{-(\lambda_1+\lambda_2)T} & \lambda_1 - \lambda_1 e^{-(\lambda_1+\lambda_2)T} \\ \lambda_2 - \lambda_2 e^{-(\lambda_1+\lambda_2)T} & \lambda_1 + \lambda_2 e^{-(\lambda_1+\lambda_2)T} \end{bmatrix} \tag{22}$$

where $T = |t_2 - t_1|$. For the scenario used in simulation, we chose $\lambda_1 = (1/20)$ s^{-1} and $\lambda_2 = (1/10)$ s^{-1}.

(ii) In the first decentralized tracking configuration both sensor 1 and sensor 2 use an IMM estimator and the fusion center fuses the local estimates every $T_F = 10$ s using the two local state estimates with the corresponding covariances. The local tracker at sensor 1 uses $\tilde{q}_l = 1$ m^2/s^3 and $\tilde{q}_h = 8000$ m^2/s^3. The local tracker at sensor 2 uses $\tilde{q}_l = 1$ m^2/s^3 and $\tilde{q}_h = 20000$ m^2/s^3.

(iii) In the second decentralized tracking configuration sensor 1 uses the same IMM estimator as in (ii) while sensor 2 uses a legacy filter (in both north and east coordinates) with a fixed filter gain. The optimal filter gain components α and β vs. the target maneuvering index are shown in Fig. 3. The values we used are $\alpha = 0.86$ and $\beta = 0.74$ which correspond to a target maneuvering index around 2. To implement the track-to-track fusion with a legacy tracker, the fusion center needs first to obtain the covariance (approximate MSE matrix) of the local state estimate from the legacy tracker. This is done according to the procedure discussed in Section 3.1. The track fusion is done with

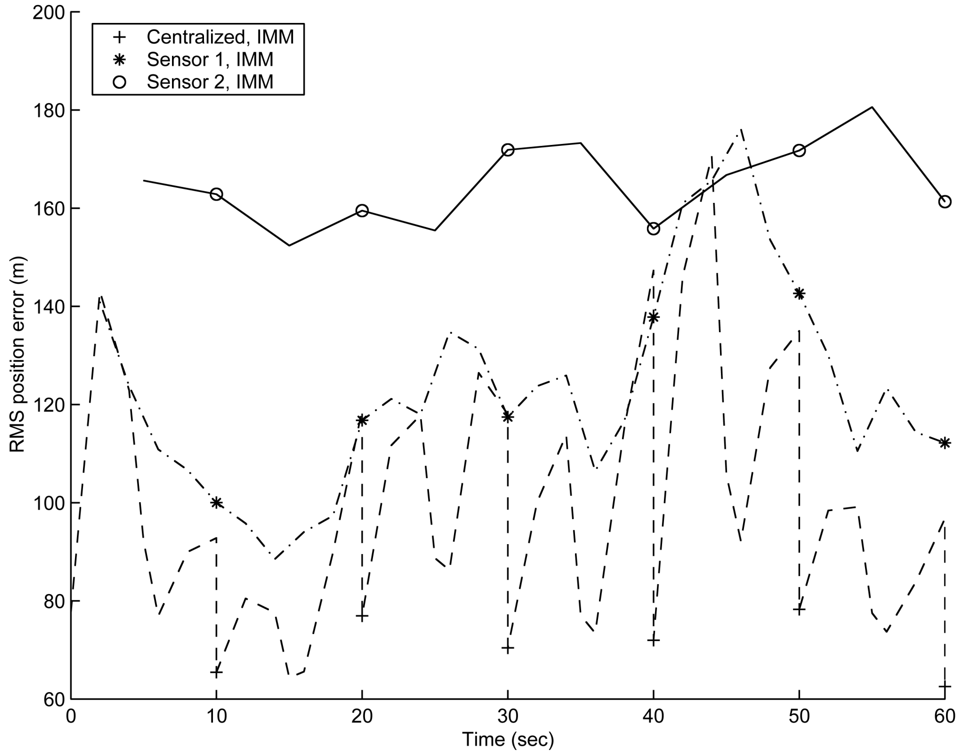

Fig. 4. RMS position errors for centralized IMM estimator vs. two local IMM estimators.

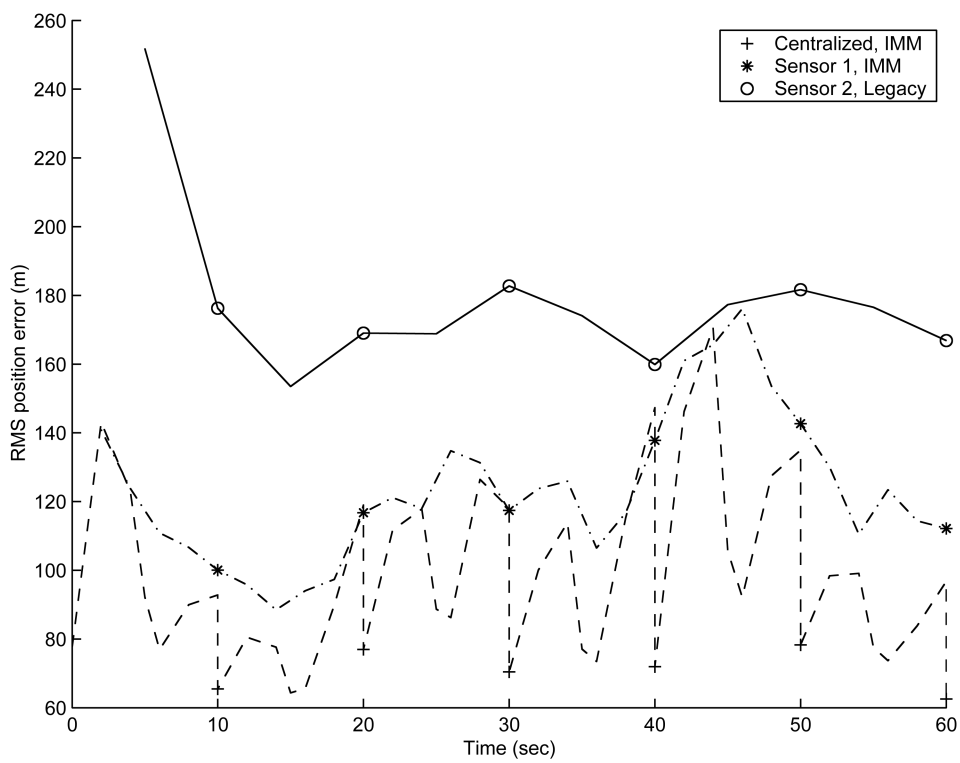

Fig. 5. RMS position errors for centralized IMM estimator vs. two local estimators (IMM and legacy).

crosscovariances calculated using the fixed crosscorrelation coefficients as in (ii).

Fig. 4 shows the root mean square (RMS) position errors of the centralized IMM estimator vs. the two local IMM estimators at sensor 1 and sensor 2 from 100 Monte Carlo runs. Special symbols indicate the times when track-to-track fusion is carried out. The local tracker at sensor 1 has better estimation accuracy in position than the local tracker at sensor 2 since it has a higher measurement rate.

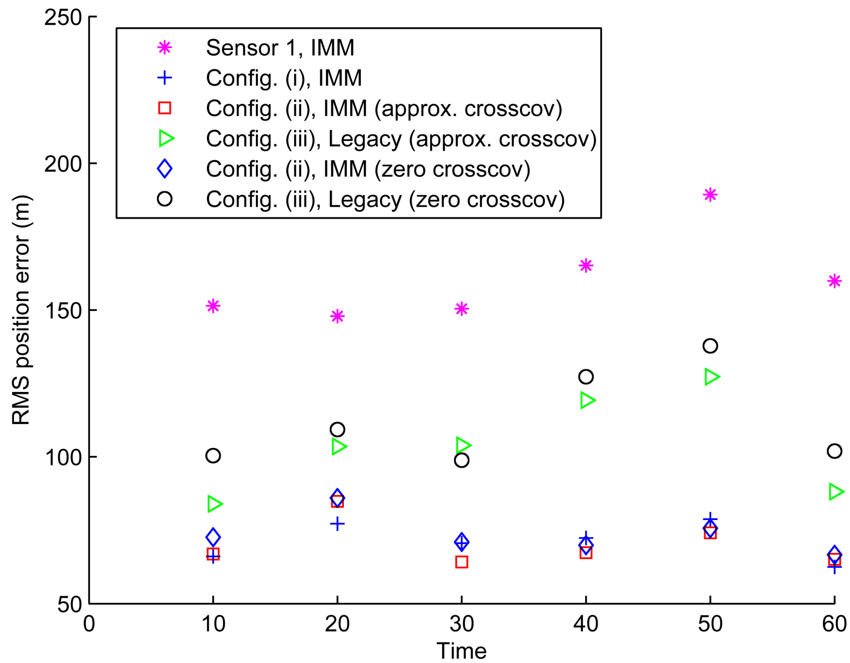

Fig. 6. Comparison of the RMS position errors for centralized IMM estimator vs. track fusion with an IMM estimator and a legacy filter.

Fig. 5 shows the RMS position errors of the centralized IMM estimator vs. two local estimators at sensor 1 and sensor 2 where the local tracker at sensor 1 uses an IMM estimator and the local tracker at sensor 2 uses a legacy filter from 100 Monte Carlo runs. Compared with Fig. 4, we can see that the performance degrades when sensor 2 uses a legacy filter rather than an IMM estimator.

Fig. 6 shows the RMS position errors at the fusion center for the above three tracking configurations as well as that by sensor 1 alone. In configurations (ii) and (iii), both approximate crosscovariance and zero crosscovariance were used in the track fusion procedure. We can see that the track fusion of two local IMM estimators has the RMS position error close to that of the centralized estimator. Assuming zero crosscovariance does not affect the position estimation accuracy by much. However, the track fusion with a legacy filter has a moderate performance gap compared with the centralized estimator for the RMS position error. We can also see that the performance of the fused estimate using a legacy track is clearly better than that using sensor 1 alone.

Fig. 7 shows the normalized estimation error squared (NEES) [2] at the fusion center for the above three tracking configurations as well as that by sensor 1 alone. The 99% percent confidence interval is also shown assuming that the NEES statistic is chi-square distributed with the appropriate degrees of freedom. We can see that nearly all fusion results are pessimistic during the non-maneuver motion segments owing to the zero process noise of the true target motion. However, configurations (ii) and (iii) yield larger NEES than the centralized estimator during the target turns, i.e., the estimation error covariance at the fusion center is more optimistic compared with that of the single sensor estimate. Assuming zero crosscovariance can make the situation even worse. Thus caution has to be exercised when fusing local estimates that are not consistent with their calculated covariances.

5. CONCLUDING REMARKS

In this paper a procedure for reconstruction of legacy trackers' state estimation error covariances was described for use in track-to-track association and fusion algorithms that account for the crosscovariance of the estimation errors between local tracks. In addition, a practical way to approximate these crosscovariances has been presented. A two-sensor tracking example, with one of the trackers being a legacy tracker, indicates the effectiveness of the resulting distributed tracking system with track fusion on demand. The performance of this system exhibits only a modest degradation compared with a centralized tracker using an interacting multiple model estimator.

APPENDIX. THE EXACT CROSSCOVARIANCE FOR ASYNCHRONOUS SENSORS

The recursion for the crosscovariance given in Eq. (8.4.2-3) of [3] is for synchronous sensors. The recursion for the case of asynchronous sensors is as follows.

Let $\{t_m^i\}_{m=1}^{N^i}$ and $\{t_n^j\}_{n=1}^{N^j}$ be the sampling times at sensor i and j, respectively. The union of these sets, with the times ordered, is denoted as

$$\mathcal{T}^{ij} \triangleq \{t_k^{ij}\}_{k=1}^{N^i+N^j}. \tag{23}$$

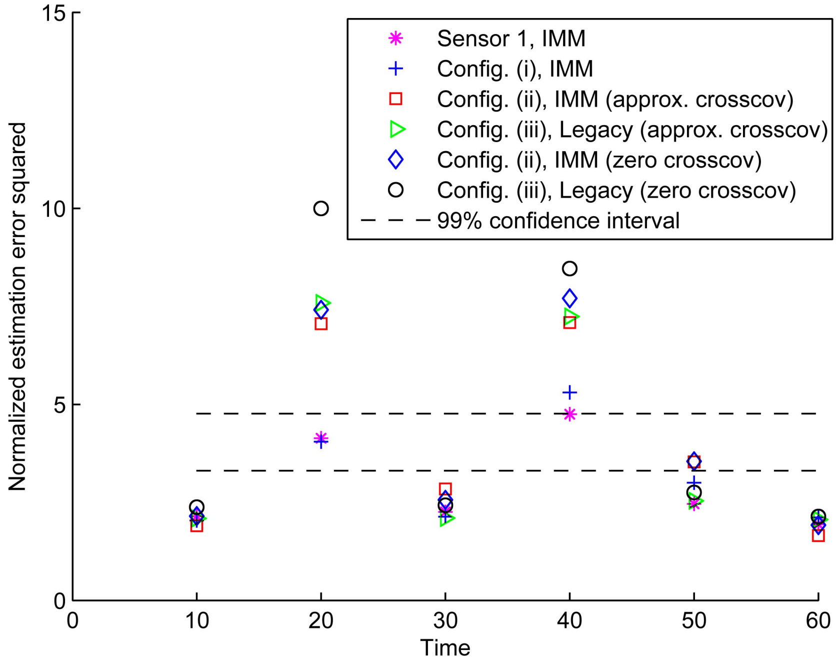

Fig. 7. Comparison of the NEES for centralized IMM estimator vs. track fusion with an IMM estimator and a legacy filter.

Then the generalized version of the crosscovariance recursion will be iterated on the ordered union set (23) as follows

$$P^{ij}(t_k^{ij}) \stackrel{\Delta}{=} E[\tilde{x}^i(t_k^{ij})\tilde{x}^j(t_k^{ij})']$$
$$= [I - W^i(t_k^{ij})H^i(t_k^{ij})][F(t_k^{ij},t_{k-1}^{ij})P^{ij}(t_k^{ij}-1)F(t_k^{ij},t_{k-1}^{ij})' + Q(t_k^{ij},t_{k-1}^{ij})][I - W^j(t_k^{ij})H^j(t_k^{ij})]' \quad (24)$$

where the estimation error $\tilde{x}$ has t_k^{ij} as its single argument indicating the current time. This error might correspond to a current estimate, or a prediction as in (17). The gain for filter i in the above is

$$W^i(t_k^{ij}) = 0 \qquad \text{if} \quad t_k^{ij} = t_n^j \quad (25)$$

i.e., it is zero at the times when only filter j carries out an update, and the other way around; $F(t_k^{ij},t_{k-1}^{ij})$ is the state transition matrix from t_{k-1}^{ij} to t_k^{ij} and $Q(t_k^{ij},t_{k-1}^{ij})$ is the covariance of the process noise over the interval $t_k^{ij} - t_{k-1}^{ij}$. The initial condition for (25) is $P^{ij}(t_1^{ij}) = 0$, assuming the filters use independent initial information.

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